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A Unified Approach for Representing Wireless Fading Channel using EM-based Mixture of Gamma Distributions

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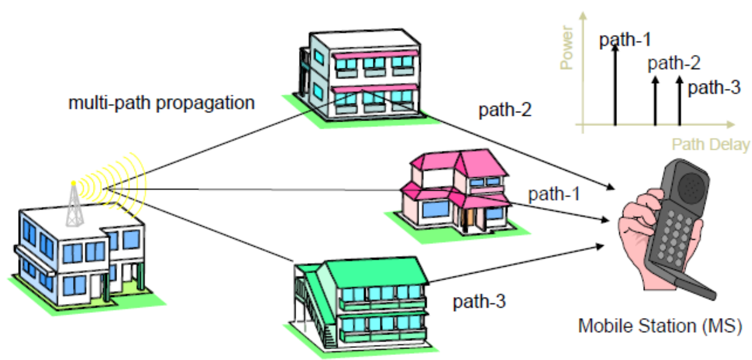
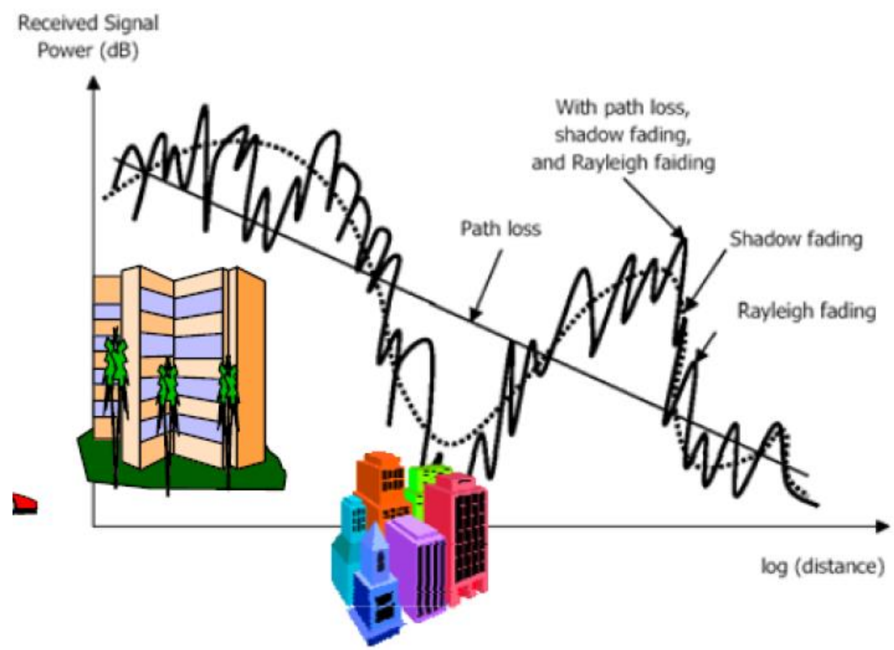
- Outline
  - Introduction | Literature Review
    - Physical Channel Modeling.
    - Composite Fading Channels.
  - Proposed Approach
    - Expectation-Maximization Algorithm.
    - Model Analysis Comparisons.
  - Performance Analysis | Fixed Gain Relaying
    - Fixed Gain Relaying Scheme over Generalized Fading.
      - Raw Moments, Amount of Fading, Capacity.
    - Simulation Results.
  - Conclusions

Outline

Physical Channel Modeling

- Path Loss
  - $P_t \propto \frac{1}{d^n}$
- Shadowing Effect
  - Lognormal Distribution
- Multipath Fading Effect
  - Rayleigh, Nakagami-m, Weibull-β
  - Rician, etc.





- Composite Fading Environments
  - We focus on Nakagami-m/Lognormal Fading.
  - Shadowing: Lognormal.
    - $f_L(\sigma) = \frac{\lambda}{\sqrt{2\pi}\sigma\zeta} e^{-\frac{(10\log_{10}\sigma - M)^2}{2\zeta^2}}$
  - Multipath: Nakagami-*m,* Rician, Weibull.
    - $f_N(\alpha|\sigma) = \frac{m^m}{\gamma_0^m \Gamma(m)} \gamma^{m-1} e^{-\frac{m\gamma}{\gamma_0}}$
  - Composite Envelope
    - $f_\alpha(\alpha) =$

$$\frac{\lambda m^m}{\Gamma(m)\sqrt{2\pi}\zeta} \int_0^\infty \frac{x^{m-1}}{\sigma^{m+1}} e^{-\frac{mx}{\sigma^2}} e^{-\frac{(10\log\sigma - M)^2}{2\zeta^2}} d\alpha$$

⚠ Intractable

Introduction | Literature Review

- Composite Fading Environments
    - Use of Moment Matching Methods.
  - K and K_G Distributions.
    - Replace Lognormal by Gamma Distributions, and solve.
    - Contains modified Bessel Function of second type, as
      - $f_\alpha(\alpha) = \frac{4}{\Gamma(v)}\left(\frac{v}{w}\right)^{\frac{v+1}{2}} \alpha^v \boldsymbol{K}_{v-1}\left(\sqrt{\frac{4v}{w}}\alpha\right)$
  - RIGD and G Distributions.
    - Replace lognormal by Inverse-Gaussian and Solve.
    - Comparable Complexity with the Former, written as
      - $f_\alpha(\alpha) = \frac{4m^{\frac{\beta+1}{2}}}{\Gamma(m)\Gamma(k)\Omega^{\frac{\beta+1}{2}}} \alpha^\beta \boldsymbol{K}_v\left(2\left(\frac{m}{\Omega}\right)^{\frac{1}{2}} \alpha\right)$

Introduction | Literature Review

- Composite Fading Environments
  - Mixture Gamma Distribution [2011, Atapattu *et al.*]
    - $f_\alpha(\alpha) =$
      $$\frac{\lambda m^m}{\Gamma(m)\sqrt{2\pi}\zeta}\int_0^\infty \frac{x^{m-1}}{\sigma^{m+1}} e^{-\frac{mx}{\sigma^2}} e^{-\frac{(10\log\sigma - M)^2}{2\zeta^2}} d\alpha$$
    - Used Gauss-Quadrature Methods (Moment Matching).
    - $f_\gamma(\gamma) = \sum_{i=1}^{C} \omega_i \left(\frac{\gamma}{\gamma_0}\right)^{\beta_i - 1} \exp\left(-\zeta_i \frac{\gamma}{\gamma_0}\right)$
    - Approximates:
      - Nakagami-m, Rayleigh, , η_μ, κ_μ, …
      - Nakagami/Lognormal, Rician.

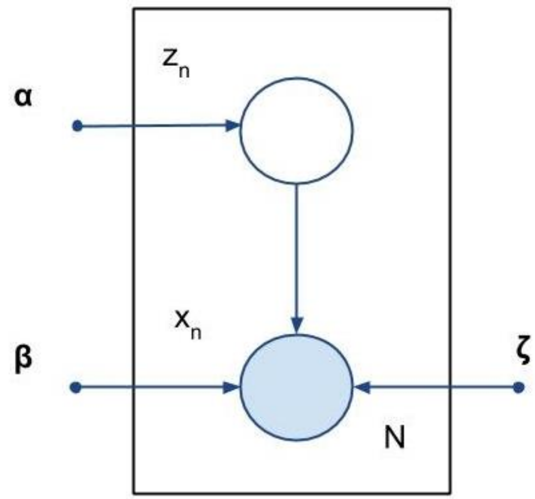
Not General for all channels

Introduction | Literature Review

- Approximation Methodology: EM Algorithm
  - Adopt **Mixture Gamma** Distribution.
  - Maximize Log-likelihood estimates via the use of **Expectation-Maximization** (EM) Algorithm.
    - Coined by Dempster *et al.* in 1977.
    - Iterative solution to intractable Maximum Likelihood Estimation (MLE) problems.
  - Pros:
    - Automated (Unsupervised Algorithm).
    - Proven not to get worse as it iterates by.
  - Cons:
    - Might get stuck in a local maxima. Though, works well in practice.

Adopted Approach | EM

- Approximation Methodology:
  - Let the j^{th} entry of an observed SNR vector $X = (x_1, x_2, \dots, x_j, \dots, x_N)$ be regarded as **incomplete** data.
  - Assume there exist hidden variable (Z), where $\Pr(z_n = i) = \alpha_i$.
  - Assume that the marginal distribution X follows a Mixture Gamma (MG) distribution
    - $f_\theta(x_j) = \sum_{i=1}^{C} \alpha_i \, \phi_i(x_j; \beta_i, \zeta_i)$
      - » $where, \phi_i(x_j; \beta_i, \zeta_i) = x^{\beta_i - 1} e^{-\zeta_i x}$
  - Goal: Maximize Log-likelihood
    - $\text{maximize} \, L(\theta) = \sum_{j=1}^{N} f_\theta(x_j)$.

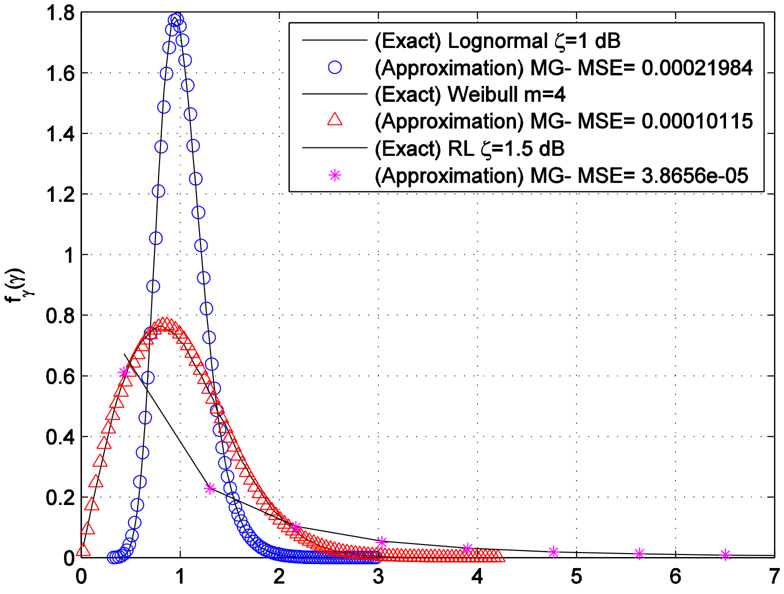


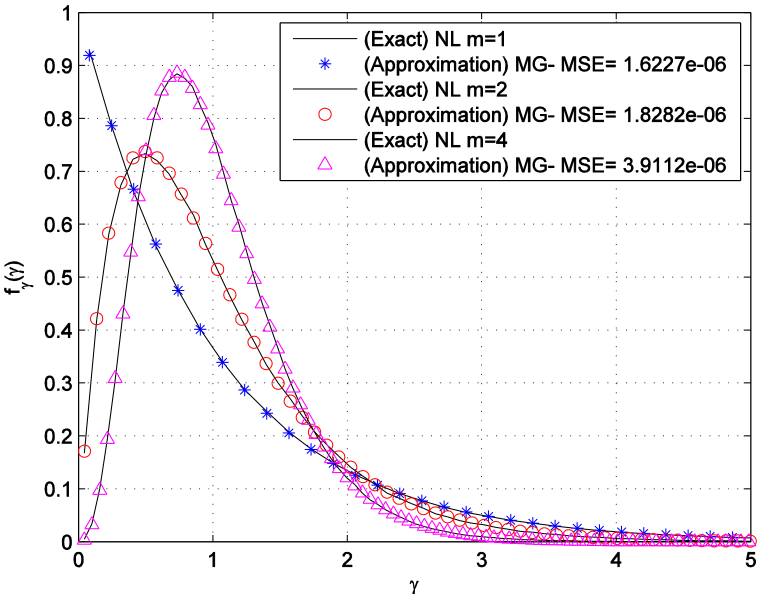
Adopted Approach | EM

- Approximation Methodology:
  - $\boldsymbol{L}(\theta)$ maximized iteratively through EM.
  - Consists of Two Main Steps:
    - Expectation Step: Compute Posterior Probability
      - $\tau_{ij}^{(t)} = \frac{\alpha_i \phi_i(x_j,\theta)}{\sum_i \alpha_i \phi_i(x_j,\theta)}$
    - Maximization Step:
      - $\alpha_i^{(t+1)} = \frac{1}{n}\sum_{j=1}^{n} \tau_{ij}^{(t)}$
      - $\theta_i^{(t+1)} = [\zeta_i, \beta_i]$ are computed by maximizing the likelihood using the Newton-Raphson algorithm, using ‘nlm’ package in Software R.
    - Iterate between Expectation and Maximization until convergence is reached, $\theta_i^{(t+1)} - \theta_i^{(t)} < \delta$.

Adopted Approach | EM

- Approximation Results
  - May be used to other fading distributions.
  - mean square error (MSE) to validate accuracy.
  - Only two components is used, $C = 2$.

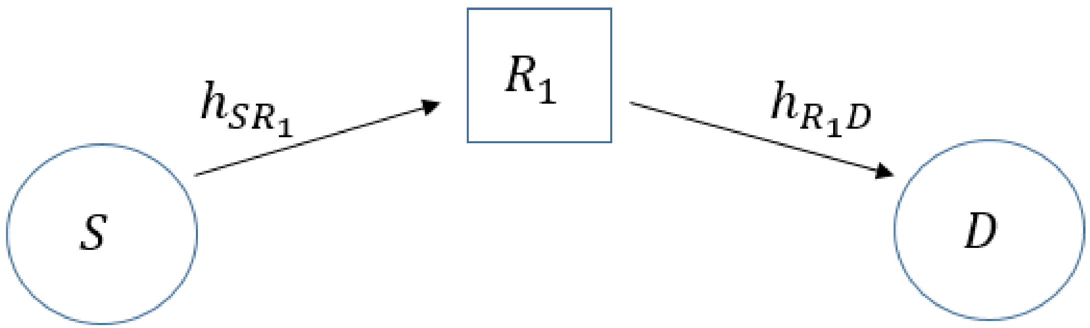




- (**Left**) Approximation of Lognormal, Weibull, and Rayleigh/lognormal. (**Right**) Nakagami/lognormal with varying severity.

Adopted Approach | Model Analysis and Comparisons

- System Model
  - Consider a dual-hop Amplify-and-Forward (AF) cooperative scenario, as shown below.
  - h_{SR_1}, h_{R_1D} are *i.n.i.d.* and follow the generalized MG distribution.
  - The relay R_1 is a fixed-gain relay, with gain G.



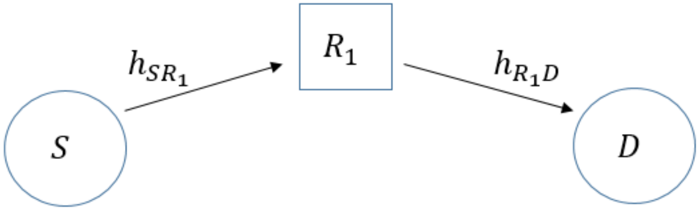
Performance Analysis | Fixed Gain Relaying

- System Model

  - $y_{SR_1}(t) = h_{SR_1}(t)x(t) + n_{SR_1}(t)$
  - $y_{R_1D}(t) = G\, h_{R_1D}(t)y_{SR_1}(t) + n_{R_1D}(t)$

– $\gamma_{end-to-end} = \frac{\gamma_{SR_1}\gamma_{R_1D}}{\gamma_{R_1D}+K}$

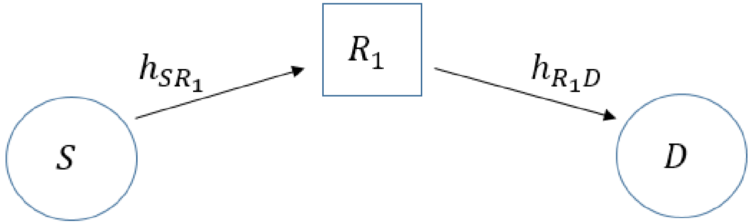
  - $K = \frac{1}{G^2 N_{01}}$ $is\ fixed\ constant.$



Current Work | Results

- Raw Moments of end-to-end SNR
  - $E\{\gamma^n\} = \int_0^\infty . \int_0^\infty \left(\frac{\gamma_1\gamma_2}{K+\gamma_2}\right)^n f_{\gamma_1}(\gamma_1) f_{\gamma_2}(\gamma_2) d\gamma_1\, d\gamma_2$
  - $f_{\gamma_i}(x) = \sum_{i=1}^{C} . \frac{\alpha_i}{\rho}\left(\frac{x}{\rho_i}\right)^{\beta_i - 1} e^{-\frac{\zeta_i x}{\rho_i}}$
  - $E\{\gamma^n\} =$

$$\sum_{j=1}^{C} \frac{\alpha_j^1 \rho_1 \Gamma\left(n+\beta_j^1\right)}{\left(\zeta_j^1\right)^{n+\beta_j^1}} \sum_{k=1}^{C} \frac{\alpha_k^2\, K^{\zeta_k^2}}{\rho_2^{\beta_k^2} \Gamma(n)} G_{2,1}^{1,2}\left(\begin{matrix} 1, 1+\beta_k^2 \\ n+\beta_k^2 \end{matrix} \middle| \frac{\rho_2}{K\zeta_k^2}\right)$$



Performance Analysis | Fixed Gain Relaying

- Amount of Fading
  - First Introduced by (Charash, 1979) as a measure of the severity of the fading channel.
  - Can be computed from first two moments.
    - $AoF = \frac{E\{\gamma_{end-to-end}^{2}\} - (E\{\gamma_{end-to-end}\})^2}{(E\{\gamma_{end-to-end}\})^2}$

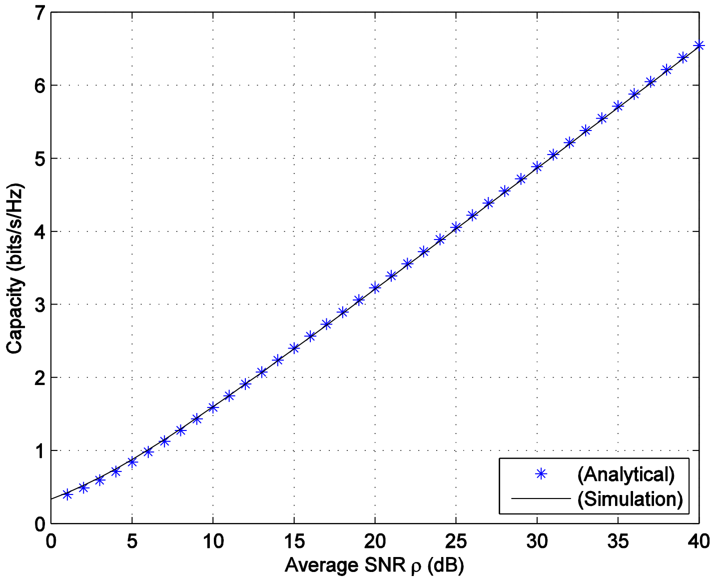
Performance Analysis | Fixed Gain Relaying

- Average Capacity
  - Defined as the mean of the instantaneous mutual information (IMI).
    - $C_{erg} = \frac{B}{\ln 2} E\{\ln(1+\gamma_{end-to-end})\}$
  - Using Taylor Series of $\ln(1+\gamma_{end-to-end})$ around $E\{\gamma_{end-to-end}\}$.
    - $\ln(1+\gamma) = \ln(1+E\{\gamma\}) + \sum_{w=1}^{\infty} \frac{(-1)^{w-1}(x-E\{\gamma\})}{w(1+E\{\gamma\})^{w}}$
    - Take Expectation of $\ln(1+\gamma)$ and truncate to first two moments.

    - $C_{erg} = \frac{B}{\ln 2} [\ln(1+E\{\gamma_{end-to-end}\}) - \frac{E\{\gamma_{end-to-end}^2\} - E^2\{\gamma_{end-to-end}\}}{2(1+E\{\gamma_{end-to-end}\})^2}$

Performance Analysis | Fixed Gain Relaying

- Average Capacity
  - h_{SR_1} follows Weibull distribution.
  - h_{R_1D} follows Nakagami-m/lognormal distribution.
  - Fixed Gain K=0.5.
  - Very accurate for whole operating SNR, with $C = 2$.



Performance Analysis | Fixed Gain Relaying

- Conclusions
  - We provided general framework to approximate any arbitrary fading distribution by presenting them as Mixture Gamma (MG) distribution.
    - Used Expectation-Maximization algorithm with Newton-Raphson maximization algorithm to maximize the log-likelihood estimates.
  - Analyze and study performance analysis of dual-hop fixed-gain AF cooperative scenario for non-composite and composite fading distributions.

Conclusion and Future Work